1.0

2.8 2.5

3.2 2.2

3.6

4.0 2.0

1.1

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ON THE PERFORMANCE OF A POLARIZATION
SENSITIVE ADAPTIVE ARRAY

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This paper considers the ability of an LMS adaptive array to adapt to the electromagnetic polarization of incoming signals. An array of two pairs of crossed dipoles is studied. A desired signal and an interference signal are assumed to arrive from arbitrary directions with arbitrary elliptical polarizations. The output signal-to-interference-plus-noise ratio (SINR) from the array is computed as a function of the signal angles of arrival and polarizations.

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   It is shown that, as long as certain special desired signal polarizations are avoided, the array is difficult to jam with a single interference signal. To produce a poor SINR, an interference signal must both arrive from the same direction and have the same polarization as the desired signal.

CONTENTS

I. INTRODUCTION

Adaptive arrays[1-3] are currently of great interest because of their ability to null interference and track desired signals automatically. Numerous papers have discussed the performance of adaptive arrays[4]. In spite of the extensive literature, however, for radio applications of these arrays (as contrasted with sonar applications), one aspect of this subject appears to have received little attention.

We refer to the fact that an adaptive array can adapt to the electromagnetic polarization of signals, as well as their arrival angles. If an adaptive array uses elements responding to more than one polarization, the array feedback loops will automatically combine the signals from these elements to optimize reception, or provide a null, for particular signal polarizations. Such an array can automatically track a desired signal with one polarization while nulling interference with a different polarization.

Most analytical studies of adaptive arrays have assumed isotropic elements. This assumption, although useful for certain purposes, tacitly eliminates any consideration of the effects of signal polarization on array performance. In essence, one assumes all signals arrive at the array with the same polarization. If an array receives and uses more than one polarization, its performance can be far superior to one that does not. For example, an array of isotropic elements always yields poor performance if interference arrives too close to the desired signal. When an array adapts to polarization, however, this difficulty occurs only if both signals have the same polarization as well as angle of arrival. When two signals arrive from the same direction, it is perfectly possible to null one signal and not the other, if their polarizations are different.

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The purpose of this paper is to examine the performance of a polarization sensitive adaptive array. As a model, we will consider an array of two pairs of crossed dipoles. We will compute the output signal-to-interference-plus-noise ratio (SINR) from this array when a desired signal and an interference signal arrive with arbitrary polarizations and angles of arrival.* We will show that in most cases interference has little effect on the array output SINR unless it arrives from the same direction and has the same polarization as the desired signal. However, there are two exceptions. If the desired signal polarization is linear, oriented either parallel or perpendicular to the vertical dipoles, the array is susceptible to interference from other angles as well. These desired signal polarizations are ones that should be avoided in a system design. Finally, we will find that when both signals arrive from broadside, the array output SINR is simply related to the separation between the signal polarizations on the Poincare sphere.

Section II of the paper formulates the necessary equations. Section III contains the calculated results and Section IV the conclusions.

II. FORMULATION OF THE PROBLEM

Consider a four-element adaptive array consisting of two pairs of crossed dipoles, as shown in Figure 1. The signal from each dipole is to be processed separately in the array. The upper and lower dipole pairs have their centers at $Z = +\frac{L}{2}$ and $Z = -\frac{L}{2}$, respectively. Let $\tilde{x}_1(t)$ and $\tilde{x}_3(t)$ be the complex signals received from the upper and lower vertical dipoles,

*By arbitrary polarizations, we refer to signals that are <u>completely</u> polarized (i.e., elliptically polarized). We do not consider <u>partially</u> polarized signals. [12]

and $\tilde{x}_2(t)$ and $\tilde{x}_4(t)$ the signals received from the upper and lower horizontal dipoles, respectively. Each signal $\tilde{x}_j(t)$ is multiplied by a complex weight w_j and summed to produce the array output. We assume the weights w_j are controlled by an LMS processor[2,5], so the steady-state weight vector, $w=(w_1, w_2, \ldots, w_4)^T$, is given by

$$w = \Phi^{-1}S \qquad (1)$$

where Φ is the covariance matrix,

$$\Phi = E\left\{X^*X^T\right\}, \qquad (2)$$

and S is the reference correlation vector

$$S = E\left\{X^*\tilde{r}(t)\right\}. \qquad (3)$$

In these equations, X is the signal vector,

$$X = (\tilde{x}_1(t), \cdots, \tilde{x}_4(t))^T, \qquad (4)$$

$\tilde{r}(t)$ is the complex reference signal* used in the adaptive array feedback[2,5], T denotes transpose, "*" complex conjugate, and $E(\cdot)$ expectation.

Assume two CW signals are incident on the array, one desired and the other interference. Let θ and ϕ denote standard polar angles, as shown in Figure 1. We assume the desired signal arrives from angular direction (θ_d, ϕ_d) and the interference from (θ_i, ϕ_i). Furthermore, each signal is assumed to have an arbitrary electromagnetic polarization. To characterize the polarization of each signal, we make the following definitions.

---

*$\tilde{r}(t)$ is called the "Desired Response" in Reference 2.

Given a TEM wave propagating into the array, we consider the polarization ellipse produced by the transverse electric field as we view the incoming wave from the coordinate origin. Note that unit vectors $\hat{\phi}$, $\hat{\theta}$, $-\hat{r}$, in that order, form a right-handed coordinate system for an incoming wave. Suppose the electric field has transverse components

$$\vec{E} = E_\phi \, \hat{\phi} + E_\theta \, \hat{\theta} \, . \tag{5}$$

(We will call E_ϕ the horizontal component and E_θ the vertical component of the field.) In general, as time progresses, E_ϕ and E_θ will describe a polarization ellipse as shown in Figure 2. Given this ellipse, we define β to be the orientation angle of the major axis of the ellipse with respect to E_ϕ, as shown in Figure 2. To eliminate ambiguities, we define β to be in the range $0 \le \beta < \pi$. We also define the ellipticity angle α to have a magnitude given by

$$\alpha = \tan^{-1} r \tag{6}$$

where r is the axial ratio:

$$r = \frac{\text{minor axis}}{\text{major axis}} \, . \tag{7}$$

In addition, α is defined positive when the electric vector rotates clockwise and negative when it rotates counterclockwise (when the incoming wave is viewed from the coordinate origin, as in Figure 2). α is always in the range $-\frac{\pi}{4} \le \alpha \le \frac{\pi}{4}$. Figure 2 depicts a situation in which α is positive.

For a given state of polarization, specified by α and β, the electric field components are given by (aside from a common phase factor)

$$E_\phi = A \cos \gamma \tag{8a}$$

$$E_\theta = A \sin \gamma \, e^{j\eta} \tag{8b}$$

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where γ and η are related to α and β by[6]

$$\cos 2\gamma = \cos 2\alpha \cos 2\beta \qquad (9a)$$

$$\tan \eta = \tan 2\alpha \csc 2\beta \quad . \qquad (9b)$$

The relationship between the four angular variables α, β, γ and η is most easily visualized by making use of the Poincare Sphere concept[6]. This technique represents the state of polarization by a point on a sphere, such as point M in Figure 3. For a given M, 2γ, 2β and 2α form the sides of a right spherical triangle, as shown. 2γ is the side of the triangle between M and a point labelled H in the figure; H is the point representing horizontal linear polarization. Side 2β extends along the equator, and side 2α is vertical, i.e., perpendicular to side 2β. The angle η in Equations (8) and (9) is the angle between sides 2γ and 2β.[*] The special case when $\alpha=0$ in Equation (6) and Figure 2 corresponds to linear polarization; in this case the point M lies on the equator. If, in addition, $\beta=0$, only E_ϕ is nonzero and the wave is horizontally polarized. This case defines the point H in Figure 3. If, instead, $\beta = \frac{\pi}{2}$, only E_θ is nonzero and the wave is vertically polarized. Point M then lies on the equator diametrically behind H. The poles of the sphere correspond to circular polarization ($\alpha = \pm 45°$), with clockwise circular polarization ($\alpha = + 45°$) at the upper pole.

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[*]These relationships are derived in Reference 6. Our definitions and notation correspond exactly to those in Reference 6 if we substitute $E_\phi \rightarrow X$, $E_\theta \rightarrow Y$, $\eta \rightarrow \phi$.
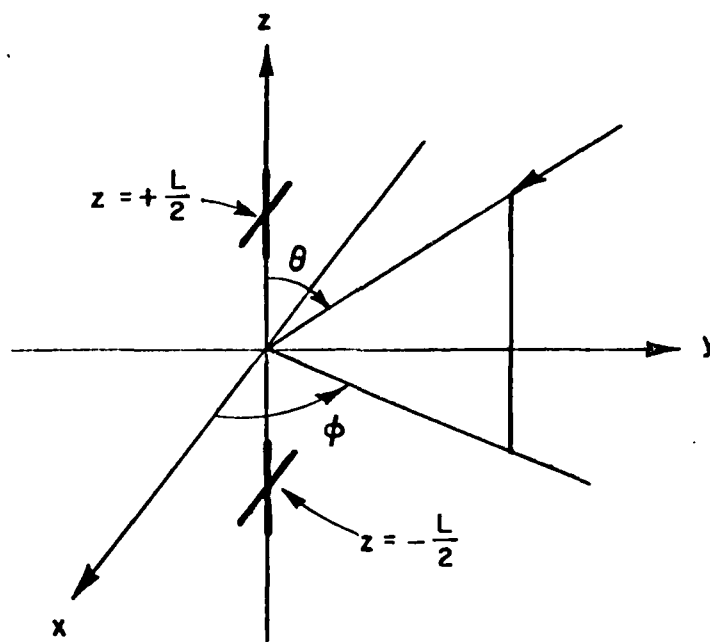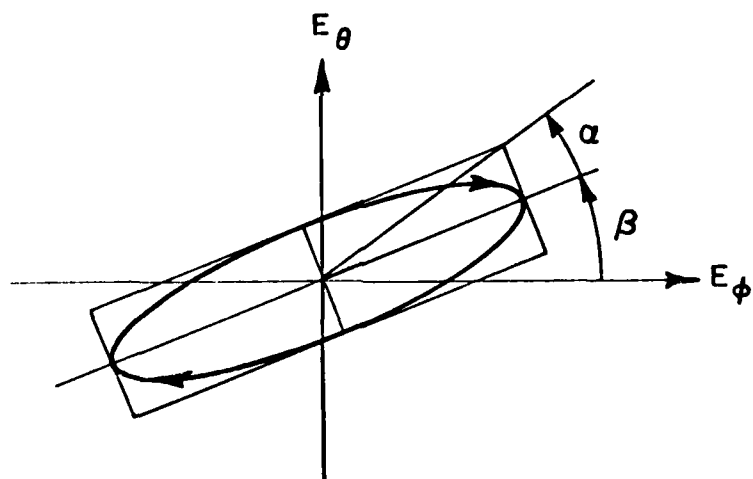
Figure 1. The crossed dipole array.
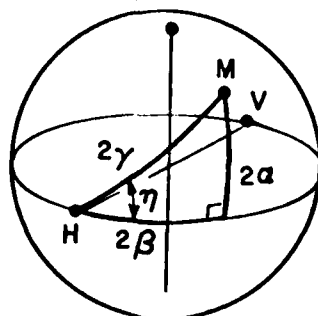

Figure 2. The polarization ellipse.

Figure 3. The Poincare sphere.

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Thus, an arbitrary plane wave coming into the array may be character-
ized by four angular parameters and an amplitude. For example, the desired
signal will be characterized by its arrival angles (θ_d, ϕ_d), its polarization
ellipticity angle α_d and orientation angle β_d, and its amplitude A_d.
(I.e., A_d is the value of A in Equation (8) for the desired signal.) We
will say the desired signal is defined by $(\theta_d, \phi_d, \alpha_d, \beta_d, A_d)$. Similarly,
the interference is defined by $(\theta_i, \phi_i, \alpha_i, \beta_i, A_i)$.

We assume each dipole in the array is a short dipole. I.e., the
output voltage from each dipole is proportional to the electric field
component along the dipole. Therefore, the vertical and horizontal dipole
outputs will be proportional to the z- and x-components, respectively,
of the electric field. An incoming signal, with arbitrary electric field
components E_ϕ and E_θ, has x,y,z components:

$$\bar{E} = E_\phi \hat{\phi} + E_\theta \hat{\theta}$$

$$= (E_\theta \cos\theta\cos\phi - E_\phi\sin\phi)\hat{x} + (E_\theta\cos\theta\sin\phi + E_\phi\cos\phi)\hat{y}$$

$$- (E_\theta\sin\theta)\hat{z} \quad . \tag{10}$$

When E_ϕ and E_θ are expressed in terms of A, γ and η as in Equation (8),
the electric field components become

$$\bar{E} = A\left[(\sin\gamma\cos\theta\cos\phi e^{j\eta} - \cos\gamma\sin\phi)\hat{x}\right.$$

$$+ (\sin\gamma\cos\theta\sin\phi e^{j\eta} + \cos\gamma\cos\phi)\hat{y}$$

$$\left. - (\sin\gamma\sin\theta e^{j\eta})\hat{z}\right] \quad . \tag{11}$$

Adding to this expression the time and space phase factors, we find that an
incoming signal characterized by $(\theta, \phi, \alpha, \beta, A)$ produces a signal vector in
the array (Equation (4)) as follows:

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$$X = A e^{j(\omega t + \psi)} U, \tag{12}$$

where U is the vector

$$U = \begin{pmatrix} (-\sin\gamma \sin\theta \, e^{j\eta}) \, e^{jp} \\ (\sin\gamma \cos\theta \cos\phi \, e^{j\eta} - \cos\gamma \sin\phi) \, e^{jp} \\ (-\sin\gamma \sin\theta \, e^{j\eta}) \, e^{-jp} \\ (\sin\gamma \cos\theta \cos\phi \, e^{j\eta} - \cos\gamma \sin\phi) \, e^{-jp} \end{pmatrix}, \tag{13}$$

ω is the frequency of the signal, ψ is the carrier phase of the signal at the coordinate origin at $t=0$, and p is the phase shift of the signals at the dipoles due to spatial delay,

$$p = \frac{\pi L}{\lambda} \cos\theta . \tag{14}$$

As stated above, we assume a desired signal specified by $(\theta_d, \phi_d, \alpha_d, \beta_d, A_d)$ and an interference signal specified by $(\theta_i, \phi_i, \alpha_i, \beta_i, A_i)$ are incident on the array. In addition we assume a thermal noise voltage $\hat{n}_j(t)$ is present on each signal $\hat{x}_j(t)$. The $\hat{n}_j(t)$ are assumed to be zero mean, to be statistically independent of each other, and to have power σ^2:

$$E\left\{ \hat{n}_i^*(t) \hat{n}_j(t) \right\} = \sigma^2 \delta_{ij}, \tag{15}$$

where δ_{ij} is the Kronecker delta.

Under these assumptions, the total signal vector is given by

$$\begin{aligned} X &= X_d + X_i + X_n \\ &= A_d e^{j(\omega t + \psi_d)} U_d + A_i e^{j(\omega t + \psi_i)} U_i + X_n, \end{aligned} \tag{16}$$

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where U_d and U_i are given by Equation (13) with appropriate subscripts d or i added to each angular quantity. ψ_d and ψ_i are assumed to be random phase angles, each uniformly distributed on $(0,2\pi)$ and statistically independent of the other. X_n is the noise vector,

$$X_n = (\hat{n}_1(t), \hat{n}_2(t) \cdots \hat{n}_4(t))^T \quad . \tag{17}$$

The covariance matrix in Equation (2) is then given by

$$\Phi = \Phi_d + \Phi_i + \Phi_n \tag{18a}$$

where

$$\Phi_d = E\left\{X_d^* X_d^T\right\} = A_d^2 U_d^* U_d^T \tag{18b}$$

$$\Phi_i = E\left\{X_i^* X_i^T\right\} = A_i^2 U_i^* U_i^T \tag{18c}$$

and

$$\Phi_n = \sigma^2 I \tag{18d}$$

with I the identity matrix.

To make the LMS array to track the desired signal, the reference signal $\hat{r}(t)$ must be a signal correlated with the desired signal and uncorrelated with the interference[2,5]. Several techniques have been described for obtaining such a reference signal[10,11]. Here we assume

$$\hat{r}(t) = A_r e^{j(\omega t + \psi_d)} \quad . \tag{19}$$

Equation (3) then yields for the reference correlation vector,

$$S = A_r A_d U_d^* \tag{20}$$

The steady-state weight vector can now be found by substituting Equations (18) and (20) into Equation (1).

The signal-to-interference-plus-noise ratio (SINR) at the array output is then given by   9

$$SINR = \frac{P_d}{P_i + P_n} \tag{21}$$

where P_d is the output desired signal power,

$$P_d = \frac{1}{2} E\left\{ |X_d^T w|^2 \right\} = \frac{A_d^2}{2} |U_d^T w|^2 , \tag{22}$$

P_i is the output interference power,

$$P_i = \frac{1}{2} E\left\{ |X_i^T w|^2 \right\} = \frac{A_i^2}{2} |U_i^T w|^2 , \tag{23}$$

and P_n is the output thermal noise power,

$$P_n = \frac{\sigma^2}{2} |w|^2 . \tag{24}$$

By making use of a matrix inversion lemma,[9,13] the expression for SINR in Equation (21) can be put in the simple form:

$$SINR = \xi_d \left[U_d^T U_d^* - \frac{|U_d^T U_i^*|^2}{\xi_i^{-1} + U_i^T U_i^*} \right] \tag{25}$$

where

$$\xi_d = \frac{A_d^2}{\sigma^2} = \text{desired signal-to-noise ratio (SNR)} \tag{26a}^*$$

$$\xi_i = \frac{A_i^2}{\sigma^2} = \text{interference-to-noise ratio (INR)} . \tag{26b}^*$$

---

*ξ_d and ξ_i are the signal-to-noise ratios that will exist in a given array element if the incoming signal arrives broadside to that element and is linearly polarized in the direction of that element. For example, if $\alpha_d = 0$ and $\beta_d = 0$, the desired signal is polarized entirely in the E_ϕ-direction. Then if the signal arrives from $\phi_d = 90^o$, the SNR on elements 2 and 4 will be ξ_d. (In this case, the SNR on elements 1 and 3 will be zero.) In general, with an arbitrary state of polarization ($\alpha_d \neq 0$ or $\beta_d \neq 0$), if the signal arrives from $\theta_d = 90^o$, $\phi_d = 90^o$ (broadside to both elements 1 and 2), the SNR on elements 1 and 2 will each be less than ξ_d. However, if the signals from elements 1 and 2 are combined with optimal weights (i.e., maximal-ratio combiner weights [7]), the total output SNR from elements 1 and 2 combined will be ξ_d. ξ_d thus represents the maximum available SNR out of each <u>pair</u> of crossed dipoles when the signal arrives broadside to both dipoles.
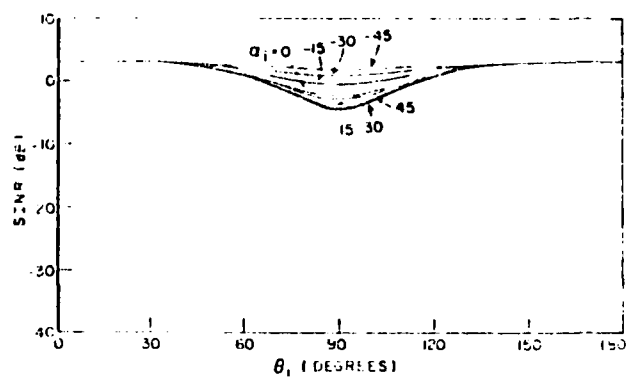
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The derivation of Equation (25) from Equation (21) may be found in the Appendix of Reference 13. Calculation of the SINR from Equation (25) is much easier than from Equations (22)-(24), because Equation (25) does not require calculation of the weight vector. In the next section, we show typical curves of the array performance based on Equation (25).
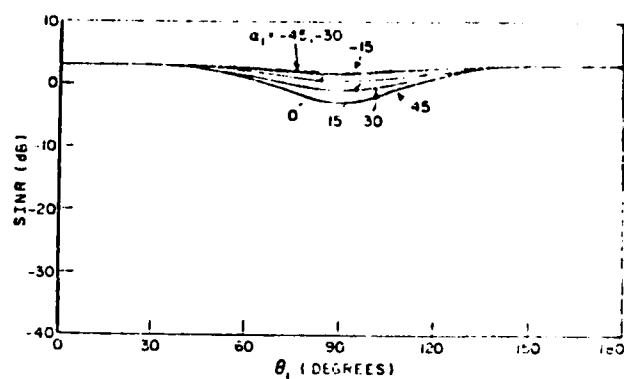
III. RESULTS

Because of the large number of parameters required to specify both the desired and interference signals, many types of curves can be plotted. Unfortunately, space does not permit an exhaustive set of curves here. However, we will show a number of typical curves, including those illustrating the worst performance.

Figures 4 and 5 show curves of output SINR when the desired signal arrives from broadside ($\theta_d = \phi_d = 90^0$). The desired signal has been chosen to have a particular elliptical polarization: $\alpha_d = 15^0$ and $\beta_d = 30^0$. The SNR is 0 dB and the INR is 40 dB. The element pairs are assumed spaced a half wavelength apart ($L = \lambda/2$). Figure 4 shows the output SINR as a function of the interference polar angle θ_i, with $\phi_i = 90^0$ and for various interference polarizations. Specifically, Figure 4a shows the SINR for $\beta_i = 0^0$, Figure 4b for $\beta_i = 30^0$, and so forth, up to Figure 4f for $\beta_i = 150^0$. Each figure shows the results for $\alpha_i = -45^0, -30^0, -15^0, 0^0, 15^0, 30^0$ and 45^0. Figure 5 shows similar results as a function of the interference azimuthal angle ϕ_i, with $\theta_i = 90^0$.
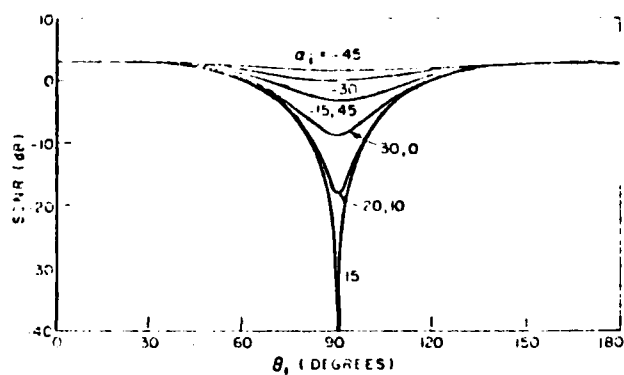
Examination of these curves shows that the worst output SINR is obtained when the interference arrives from the same direction as the desired signal (broadside) and has the same polarization as the desired signal.
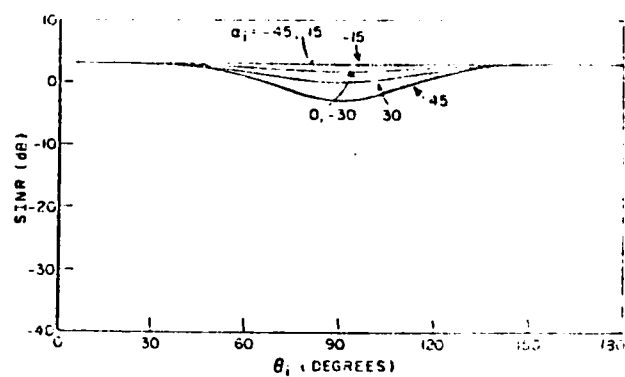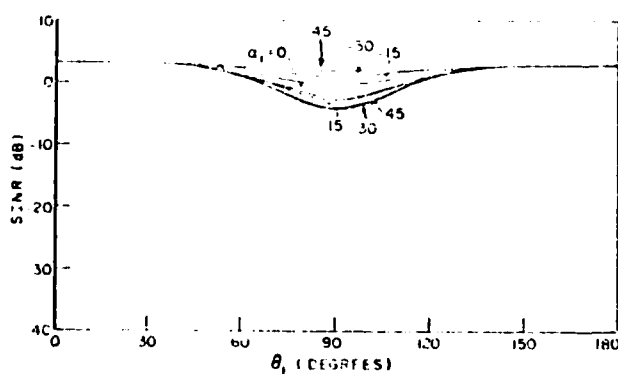
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Figure 4. SINR vs. θ_i.
$\theta_d=90^0$, $\phi_d=90^0$, $\alpha_d=15^0$, $\beta_d=30^0$, SNR=0 dB.
$\phi_i=90^0$, INR=40 dB.

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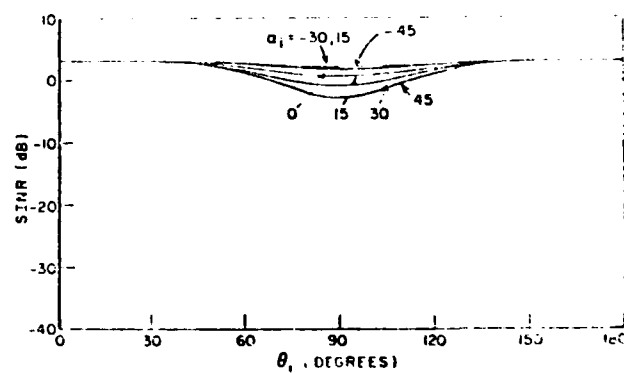
Figure 5. SINR vs. ϕ_i.
$\theta_d=90°$, $\phi_d=90°$, $\alpha_d=15°$, $\beta_d=30°$, SNR=0 dB,
$\theta_i=90°$, INR=40 dB.

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This result is not surprising, of course, because in this case when the array nulls the interference it also nulls the desired signal. However, the interesting thing about this case is how little difference in polarization between the signals is required to allow the array to provide substantial protection. For example, it may be seen in Figures 4b or 5b (for $\beta_i = \beta_d = 30^\circ$) that when $\theta_i = 90^\circ$ and $\phi_i = 90^\circ$, if either $\alpha_i = 0$ or $\alpha_i = 30^\circ$ (i.e., if α_i differs from $\alpha_d = 15^\circ$ by $\pm 15^\circ$) the SINR out of the array is higher than -9 dB. Thus, with this small difference in polarization, the array can provide over 31 dB of protection against the interference.

For the special case where both signals arrive from broadside, the output SINR from the array can be simply related to the polarizations of the two signals. If

$$\theta_d = \phi_d = \theta_i = \phi_i = 90^\circ \tag{27}$$

Equation (13) yields

$$U_d^T U_d^* = U_i^T U_i^* = 2, \tag{28}$$

and also

$$\left| U_d^T U_i^* \right|^2 = 4 \left| \cos\gamma_d \cos\gamma_i + \sin\gamma_d \sin\gamma_i e^{j(\eta_d - \eta_i)} \right|^2$$

$$= 2 \left[1 + \cos 2\gamma_d \cos 2\gamma_i + \sin 2\gamma_d \sin 2\gamma_i \cos(\eta_d - \eta_i) \right] . \tag{29}$$

Let M_d and M_i be points on the Poincare sphere representing the polarizations of the desired and interference signals, respectively. Then, in Equation (29), $2\gamma_d$, $2\gamma_i$ and the arc $M_d M_i$ form the sides of a spherical triangle, as shown in Figure 6. The angle $\eta_d - \eta_i$ is the angle opposite side $M_d M_i$. Using a well-known spherical trigonometric identity[8], we have

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$$\cos 2\gamma_d \cos 2\gamma_i + \sin 2\gamma_d \sin 2\gamma_i \cos(\eta_d - \eta_i) = \cos(M_d M_i), \tag{30}$$

so Equation (29) can be written

$$\left| U_d^T U_i^* \right|^2 = 2\left[1 + \cos(M_d M_i) \right] = 4\cos^2\left(\frac{M_d M_i}{2}\right), \tag{31}$$

and Equation (25) becomes

$$SINR = \xi_d \left[2 - \frac{4\cos^2\left(\frac{M_d M_i}{2}\right)}{\xi_i^{-1} + 2} \right] . \tag{32}$$

If $\xi_i^{-1} \ll 2$, this result may be approximated by

$$SINR = \frac{\xi_d}{\xi_i} \left[1 + 2\xi_i \sin^2\left(\frac{M_d M_i}{2}\right) \right] . \tag{33}$$

These formulas show that the SINR obtained from the array when both signals are at broadside depends only on the separation between the two points M_d and M_i on the Poincare sphere.

A plot of the SINR versus the spherical distance $M_d M_i$ in angular measure, as obtained from Equation (32), is shown in Figure 7 for SNR = 0 dB and INR = 40 dB. It is seen, for example, that a separation of $M_d M_i = 26^\circ$ on the Poincare sphere results in SINR = -10 dB, an improvement of 30 dB over what it would be without the array.[*] This result holds regardless of the specific polarizations of the signals, so long as they are separated by 26° on the Poincare sphere.

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[*]To reconcile the curve in Figure 7 with the results in Figures 4b and 5b, one must note that point M in Figure 3 lies above the equator by an angle 2α. Thus, for example, a separation of $M_d M_i = 26^\circ$ corresponds to a difference of only $|\alpha_d - \alpha_i| = 13^\circ$, if $\beta_i = \beta_d$.
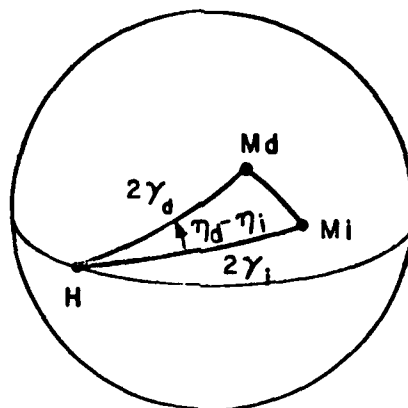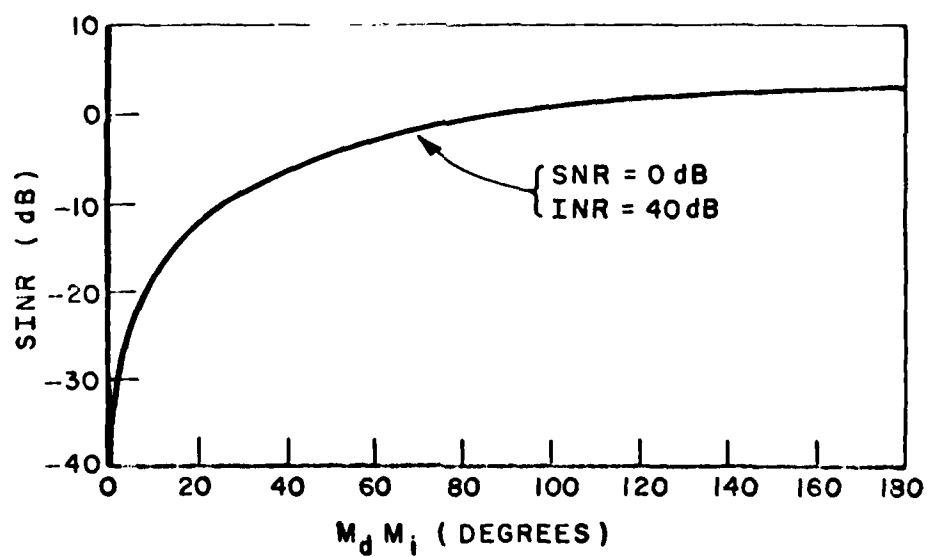
Figure 6. The points M_d and M_i.

Figure 7. SINR vs. Poincare sphere separation.
$\theta_d = \phi_d = \theta_i = \phi_i = 90^\circ$.

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In general, when the desired signal arrives from some direction other than broadside, the curves of SINR versus interference arrival angle are similar to those in Figures 4 and 5. The worst performance always occurs when the interference arrives from the same direction as the desired signal and has the same polarization. When both signals arrive from the same direction off broadside, however, it is found that the SINR cannot be related to the polarization difference so simply as in Equation (32).[*] In this case, the SINR depends on the angle of arrival as well as the polarizations. The reason is that the electric field component in the y-direction is not received by the array, because the array contains only x- and z-oriented dipoles. When the arrival angle and polarization of a signal are such that there is a y-component of electric field, the SINR is affected by the loss of power in this component to the receiving system. The amount of electric field in the y-direction depends on the angle of arrival as well as the polarization.

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[*]When the signals arrive from the same direction off broadside with different polarizations, one can express the output SINR in a form similar to Equation (32) by means of an artifice, as follows. For each signal, one defines an "equivalent signal" whose amplitude, phase and polarization are chosen to make the equivalent signal produce the same voltages in a pair of imaginary crossed dipoles oriented perpendicular to the arrival angle as the voltages produced by the actual signal in the actual dipoles. If two such equivalent signals are defined, one for the desired signal and one for the interference, the output SINR will be related to the difference in polarization of the two equivalent signals as in Equation (32). However, when this procedure is carried out, it is found that the transformation equations between each signal and its equivalent are complicated enough that little additional insight is gained. For calculating SINR, it appears to be simpler just to use Equation (25).

The curves in Figures 4 and 5 show typical performance from the array for an arbitrarily polarized desired signal. However, it must be noted that with this array certain desired signal polarizations allow the system to be jammed over a wide range of interference angles. Namely, if the desired signal excites only two of the four dipoles, then when the inter- ference excites only the same two dipoles, the array has no ability to discriminate between signals in the azimuthal coordinate ϕ. This situation leaves the array vulnerable to interference from a wide range of angles.

Specifically, there are two cases where poor performance occurs: when the desired signal is either vertically or horizontally polarized. For example, suppose the desired signal is vertically polarized ($\alpha_d = 0^\circ$, $\beta_d = 90^\circ$) and arrives from an arbitrary direction θ_d, ϕ_d. Then a vertically polarized interference signal ($\alpha_i = 0^\circ$, $\beta_i = 90^\circ$) will produce a poor SINR as long as it arrives from the same polar angle, i.e., if $\theta_i = \theta_d$, regardless of ϕ_i. It is clear from the arrangement of elements in Figure 1 why this is so. For vertically polarized signals, the array has no ability to discriminate in the azimuthal coordinate ϕ.

A particularly bad case occurs when $\theta_d = 90^\circ$ (and ϕ_d has any value). In this case, any interference signal with a nonzero vertical component (i.e., all polarizations except the case where both $\alpha_i = 0^\circ$ and $\beta_i = 0^\circ$) arriving from the particular direction $\theta_i = 90^\circ$, $\phi_i = 0^\circ$ or 180° will cause a poor SINR. The reason is that the horizontal component of a signal from $\theta_i = 90^\circ$, $\phi_i = 0^\circ$ is not received by the array, so it cannot be used to cancel interference received by the vertical elements.

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Figure 8 shows calculations illustrating this situation. In these curves, the desired signal arrives from broadside and is linearly polarized in the vertical direction. The interference arrives from $\theta_i = 90^0$ and is also linearly polarized ($\alpha_i = 0^0$). The curves show the output SINR versus ϕ_i for various interference orientation angles β_i. Two special cases should be noted. First, when $\beta_i = 90^0$, the interference is vertically polarized, and the output SINR is -40 dB regardless of ϕ_i. (The reason is obvious from the array geometry in Figure 1.) Second, when $\beta_i = 0^0$, the interference is horizontally polarized and it has no effect on the SINR for any ϕ_i. The SINR is always 3 dB in this case. (The interference is eliminated by the array by turning off the horizontal dipoles.) Between these two limiting cases, one obtains a wide variation in SINR, depending on ϕ_i and β_i. The important point here is to note how little the interference polarization has to differ from horizontal to jam the array quite effectively at the particular angle $\phi_i = 0^0$ or 180^0. For example, if β_i is only 5^0, the SINR is already down to -18.8 dB. To reiterate, a vertically polarized desired signal at $\theta_d = 90^0$ (and any ϕ_d) is particularly vulnerable to interference from the angle $\theta_i = 90^0$, $\phi_i = 0^0$ or 180^0.

The second case where the array performance is poor is when the desired signal has horizontal linear polarization. A horizontally polarized desired signal ($\alpha_d = 0^0$, $\beta_d = 0^0$) at any θ_d, ϕ_d will be interfered with by a horizontally polarized interference signal with the same polar angle, $\theta_i = \theta_d$, and any ϕ_i. The only exception occurs when the interference is near $\phi_i = 0^0$ or $\phi_i = 180^0$, where a horizontally polarized signal is not received.
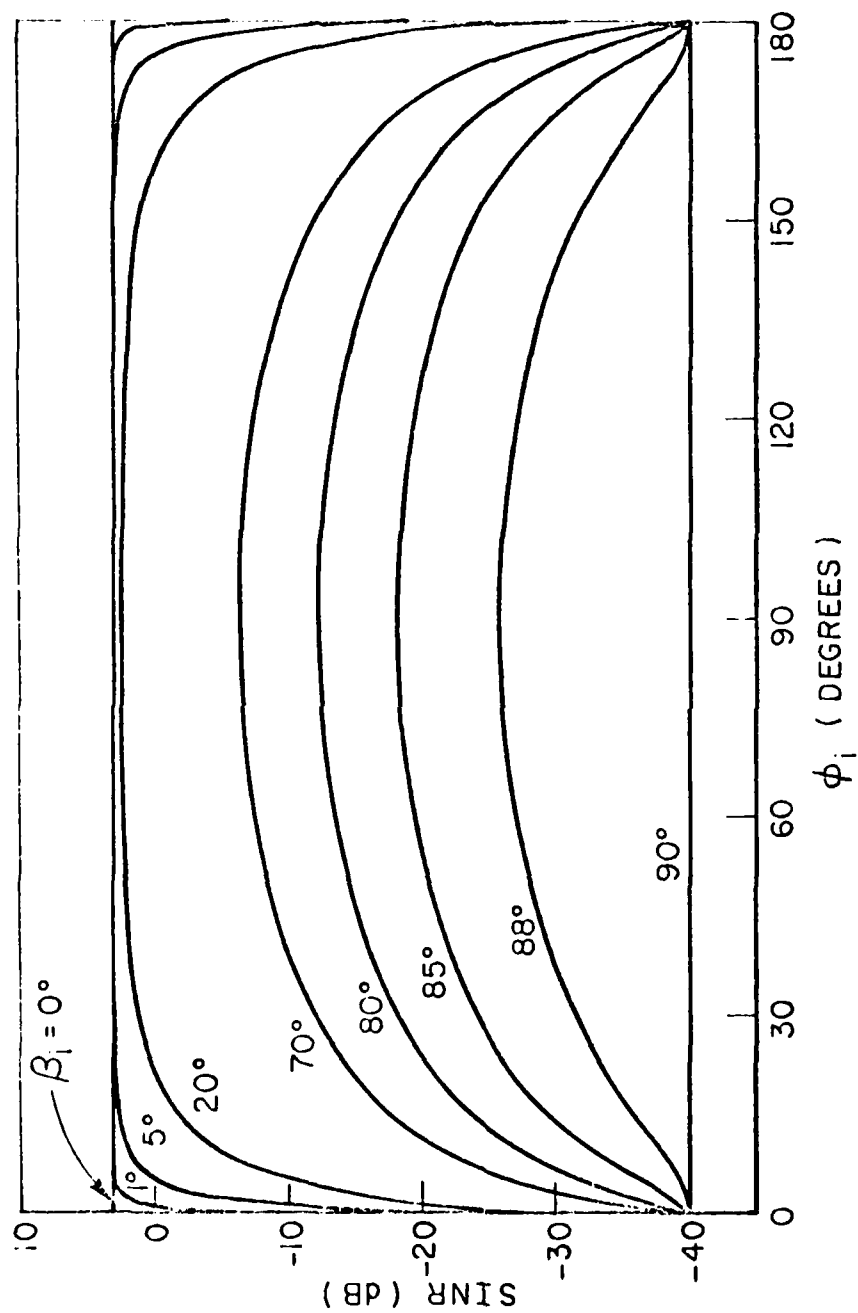
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Figure 8. SINR vs. ϕ_i.
$\theta_d=90^0$, $\phi_d=90^0$, $\alpha_d=0^0$, $\beta_d=90^0$, SNR=0 dB.
$\theta_i=90^0$, $\alpha_i=0^0$, INR=40 dB.

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Figure 9 illustrates this situation. Here the desired signal is again at broadside ($\theta_d = 90^O$, $\phi_d = 90^O$) with horizontal linear polarization ($\alpha_d = \beta_d = 0^O$). The curves show SINR versus ϕ_i for an interfering signal at $\theta_i = 90^O$ with $\beta_i = 0^O$. The different curves are for different α_i between -45° and $+45^O$. In the case $\alpha_i = 0^O$, the interference has horizontal linear polarization, and it produces a poor SINR over a very wide range of ϕ_i. Only for ϕ_i near 0° or 180° does the SINR rise, because for these angles the interference is not received by the array.

Figures 8 and 9 illustrate the vulnerability of the system to jamming when both the desired signal and the interference have either vertical linear or horizontal linear polarization. This problem occurs because in these cases the signals excite only two of the four elements and also because the dipoles in Figure 1 are all located at $x = 0$. That is, there is no displacement of the dipoles along the x-axis and hence the array has poor ability to provide spatial discrimination in the ϕ-coordinate. If we wish to discriminate well against interference that arrives with $\theta_i = \theta_d$ but with arbitrary ϕ_i, we must do one of two things: either avoid using a vertical or horizontal linearly polarized desired signal, or add more elements that can provide azimuthal discrimination, such as another dipole pair at $x = \frac{\lambda}{2}$, $y = z = 0$.

If the desired signal polarization is chosen to be something besides vertical or horizontal linear polarization, so the problems above are avoided, then the crossed dipole array is quite effective in rejecting interference. If used in a communication system, for example, this array will be rather difficult to jam with a single interference signal. Not only must the jammer arrive from the same direction as the desired signal,
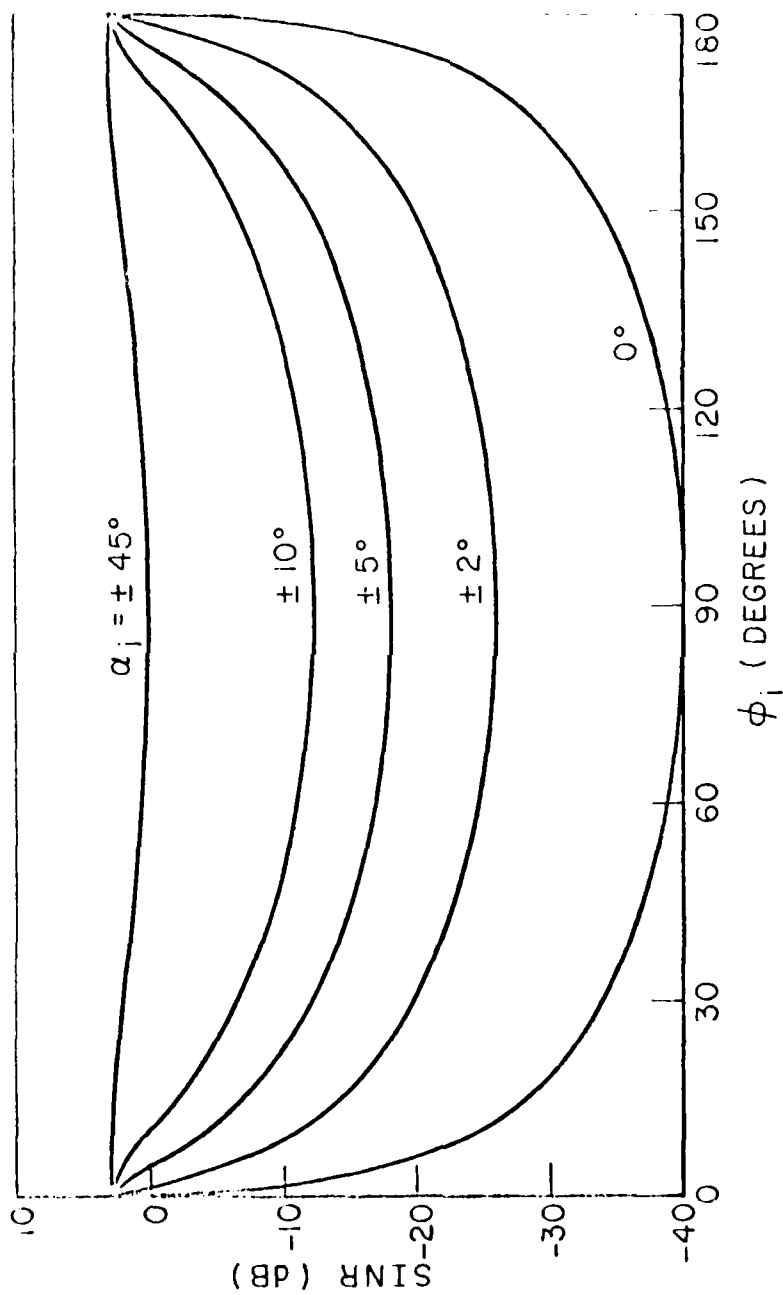
21

Figure 9. SINR vs. ϕ_i.
$\theta_d = 90^\circ$, $\phi_d = 90^\circ$, $\alpha_d = 0^\circ$, $\beta_d = 0^\circ$, SNR=0 dB.
$\theta_i = 90^\circ$, $\beta_i = 0^\circ$, INR=40 dB.

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it must have the same polarization. If the desired signal polarization is unknown to the jammer, the jammer will have to measure this polarization and modify its own polarization to match it, before it can interfere effectively. If the desired signal polarization is made to change with time*, the jammer will have to monitor desired signal polarization continuously. Moreover, the jammer must make its measurement of desired signal polarization from a location directly between the desired signal transmitter and the adaptive receiving array. Otherwise, the jammer will measure the wrong polarization, since an antenna transmitting an elliptically polarized signal usually transmits a different polarization in every direction.

IV. CONCLUSIONS

In this paper we have examined the performance of the crossed dipole adaptive array in Figure 1 with arbitrarily polarized signals. Equation (25) has been used to compute the array output SINR for a desired signal and an interference signal arriving with arbitrary angles of incidence and polarizations. Figures 4 and 5 show typical results for the case where the desired signal arrives from broadside ($\theta_d = \phi_d = 90^0$) with elliptical polarization defined by $\alpha_d = 15^0$, $\beta_d = 30^0$ (see Figure 2). In general, we find that the interference has only minimal effect on output SINR unless it arrives from the same direction and has the same polarization as the

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*A time-changing desired signal polarization is no problem for the adaptive array, because the array automatically adapts to whatever polarization it receives. (However, the speed of response of the array must be fast enough to track the rate of change of the polarization.)

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desired signal. Furthermore, when both desired signal and interference arrive from broadside, the output SINR depends only on the difference in polarization of the two signals, according to Equation (32).

Finally, we have found that certain choices for desired signal polarization lead to poor ability of the array to reject interference. Specifically, if the desired signal is linearly polarized with its electric field entirely in the θ- or ϕ-direction, the array will be vulnerable to similarly polarized interference from a wide range of angles. These types of desired signal polarization should be avoided with this array.

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